ANALYSIS OF LOAD DEPENDENT DYNAMIC TRANSMISSION ERROR RESPONSE OF GEARS WITH RANDOM PITCH ERROR

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ABSTRACT

For more realistic gear dynamic behaviour predictions, detailed gear dynamic models are needed, allowing taking into consideration the most important influencing factors. System model is presented, based on the separate handling of individual tooth pairs, with their specific profile corrections, manufacturing errors etc. Further on, non-linear single tooth pair force-deflection curve is considered, resulting in load dependent eigenfrequency characteristics. Simulation results are presented for gears with randomly distributed pitch errors. Gears with normal tooth profile and with tip relief are compared, and vibration response characteristics are analysed based on Fourier analysis of simulated transmission error response curves.

KEYWORDS: gear dynamics, random error, transmission error, Fourier analysis

1 INTRODUCTION

For more realistic vibration characteristics predictions of transmission systems especially in cases with important outer load variations, as e.g. in transport means transmissions, simplified rheo-linear models, supposing ideal involute tooth profiles and ideal mesh conditions, resulting in a linear system behaviour [1], are not always satisfactory. As experiments and theoretical considerations show, tooth geometry specialities, as tip relief [7], one of the most effective tools for dynamic behaviour improvement, real mesh conditions and mesh irregularities, influenced considerably by manufacturing errors, (manufacturing errors mean in this context, the tolerated deviations of real dimensions from the theoretical one) are to take into consideration. All these parameters, influencing considerably frequency composition of the inner excitation effects of gears, generate load dependent parametric excitation, so strongly non-linear system response develops [2,3].

Taking into account further, that real manufacturing errors are of random character, models are needed, permitting the simulation of the rolling down of more realistic gears with random errors. The results of the dynamic simulations, as dynamic force, or transmission error [7] versus time functions in that case, can be considered as quasi-random function realisations, and can be handled with statistical tools. Further on, inner excitation function model for gears with random pitch error, simulated transmission error curves for different nominal load values will be presented and their frequency characteristics will be analysed and compared for normal toothing and with adequate tip relief.

2 NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$F_{Nj}$</td>
<td>normal force on unit width of $j^{th}$ tooth pair</td>
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<tr>
<td>$F_N$</td>
<td>normal mesh force on unit width</td>
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<tr>
<td>$J_1, J_2$</td>
<td>moment of inertia of pinion and gear respectively</td>
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<tr>
<td>$K_j$</td>
<td>damping coefficient of $j^{th}$ tooth pair</td>
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<tr>
<td>$K_D$</td>
<td>dynamic factor for tooth mesh force</td>
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<tr>
<td>$N$</td>
<td>frequency ratio</td>
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<tr>
<td>$T_1, T_2$</td>
<td>torque on pinion and gear respectively</td>
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<tr>
<td>$b$</td>
<td>gear width</td>
</tr>
<tr>
<td>$h$</td>
<td>backlash</td>
</tr>
<tr>
<td>$f_z$</td>
<td>tooth frequency</td>
</tr>
<tr>
<td>$m$</td>
<td>module [mm]</td>
</tr>
<tr>
<td>$n_1$</td>
<td>number of rotation, pinion</td>
</tr>
<tr>
<td>$r_{b1}, r_{b2}$</td>
<td>base circle radius of pinion and gear respectively</td>
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3 EXCITATION FUNCTION MODEL

The inner excitation of the gears is due to the departure of the real tooth flanks from the theoretical ones on one side, and to the mesh stiffness variation during rolling down, on the other side. The first one, the kinematic excitation is caused by intended profile modifications, manufacturing errors, involving load dependent mesh irregularities, whilst the second one, the dynamic excitation is originated from the stiffness variation of the tooth pairs in contact, from the non-linear tooth deflection curves [5] and the alterations of the number of teeth, being instantaneously in contact. As a result, non-uniform rotation transmission is realised, i.e. transmission error develops.

For the description of kinematic excitation component, let us consider a tooth pair, manufactured with a resultant base pitch error \( f_{pbrj} \), Fig. 1. Assuming the pinion in fixed angular position \( \phi_1 \), for taking into contact the tooth flanks, wheel should be turned back to angular position \( \phi_2 \) (neighbouring teeth imagined as removed), from its nominal \( \phi_2n \) position, with angle difference \( \Delta \phi_2 \). For the \( j \)th tooth flank pair, contact function \( \delta_j(\phi_1) = r_{fbrj}(\phi_2n - \phi_2) > 0 \) gives the travel error, measured on the pressure line, for involute profiles, in case of contact of the given profile pair. So, for each individual profile pair combinations, realised during the rolling down on angular interval \( \Omega = 2\pi z \), giving different combinations [1], individual contact functions can be defined, Fig 2. The curved parts on \( \delta_j \) contact functions for normal profiles Fig. 2.a., belong to the mesh zones in the case of leaving later the contact and entering earlier into it. These irregular mesh zones are originated to tooth deflections under load or/and manufacturing errors. On Fig. 2.b. contact function series is represented for flanks with tip relief, showing inclined straight segments for involute relief profile. Other individual tooth error types can be modelled in a similar way [4]. Assuming the manufacturing errors for each
profile to be known, one can set the \( \delta \) functions, different for each different profile pair combinations. For zero load, the kinematic excitation function \( \delta(\phi) \) is \( \Omega \) periodic, and one can select the starting point for rolling down so, that \( \delta(\phi) > 0 \) for all \( \phi \), hence \( \delta(\phi) > 0 \) as well. The \( \Delta \sigma_1(\phi_1):F_N = r_{b1} \cdot \phi_1 + r_{b2} \cdot \phi_2(\phi) \) transmission error at \( \Delta \sigma_1 \geq 0 \) can be expressed generally as the sum of kinematic error and tooth deflection, \( \Delta \sigma = \delta_{(i+1)}(\phi_i) + w_{(i+1)} \), see Fig. 3.b. Hence, the transmission error, besides tooth deflection, tooth profile modifications, manufacturing errors, is influenced by load dependent mesh conditions as well.

Making use of the individual handling of tooth pairs being actually in contact, for the integrated treatment of the different types of excitation components, i.e. resulting in travel error, one can introduce the two variables reduced stiffness function, defined as:

\[
\hat{s}(\phi; \Delta \sigma) = \begin{cases} 
0 & \text{if } \Delta \sigma \leq \delta(\phi_i) \\
\frac{\sum F \hat{s}}{\Delta \sigma} - \frac{\sum \hat{s}(\phi) w_j}{\Delta \sigma} & \text{if } \Delta \sigma > \delta(\phi_i)
\end{cases}
\]

being a generalisation of the mesh spring stiffness function \( s(\phi_i; \Delta \sigma) \), defined for ideal contact (without irregular mesh zones), of ideal toothing, assuming non-linear tooth deflection curve [1]. In Eq. (1) \( j \) stands for the index number of profile pairs being actually in contact. In case of non-linear tooth deflection characteristics [5], Eq. (1) become more complicated, but essentially remains similar. On Fig. 3.a. reduced stiffness function is schematically presented, for normal and for reversed torque transmission with marked symbols, i.e. lower part. Reduced stiffness function curve to constant mesh load can be obtained, as the intersection of surface \( \hat{s}(\phi; \Delta \sigma) \); \( \Delta \sigma = F_N = \text{const.} \), with surface \( \hat{s}(\phi; \Delta \sigma) \).

In general case, the reduced stiffness function can be written as the sum of its Fourier components \( C_j \), with the \( C_0 \) average value as follows:

\[
\hat{s}(\phi; \Delta \sigma) = C_0(\Delta \sigma) + \sum_{k=1} C_k(\Delta \sigma) \cos \left( \frac{2\pi}{\Omega} k \phi + \nu_k \right)
\]

where \( \Omega \) is the basic angular period of the reduced stiffness function, \( k \) is the ordinal number of the Fourier components, and \( \nu_k \) the phase angle. In the case of toothing with tip relief or with manufacturing error or/and with non-linear single tooth pair force-deflection curve, values of \( C_0 \), \( C_k \) are load (i.e. \( \Delta \sigma \)) dependent.

4 SYSTEM DYNAMIC MODEL

For the study of the effect of toothing on transmission error characteristics, a simple two mass model was applied, for avoiding the disturbances of other elements as shafts, bearings etc. According to the modelling of excitation function described in chapter 3, the two rotating masses are coupled with a spring system, composed from individual tooth pairs, modelled with parallel connected spring and damping elements, completed with kinematic excitation components \( \delta(\phi) \) and \( \delta(\phi) \) respectively, Fig. 4., where symbols with mark stand for the reversed torque transmission. Hence, the system of differential equations of motion, containing the reduced stiffness term (Eq. (1)) will be as follows:

\[
J_1 \ddot{\phi}_1 + \left[ \sum K_j(\Delta \sigma - \delta_j(\phi_1)) \right] r_{b1} + r_{b2} \hat{s}(\phi_1; \Delta \sigma) \Delta \sigma = T_1
\]

\[
J_2 \ddot{\phi}_2 + \left[ \sum K_j(\Delta \sigma - \delta_j(\phi_2)) \right] r_{b2} + r_{b2} \hat{s}(\phi_2; \Delta \sigma) \Delta \sigma = -T_2
\]

The above equation, containing the \( \hat{s}(\phi; \Delta \sigma) \) load dependent excitation term, as a rheo-nonlinear system of differential equations, results parametric vibrations with resonance locations governed by system eigenfrequency, \( \omega_1 \). The eigenfrequency is determined by \( s(\phi_i; \Delta \sigma) \) load dependent mesh spring stiffness function in case of non-linear tooth deflection characteristics [5], as a generalisation of the \( s(\phi_i) \) stiffness function, belonging to constant tooth stiffness. The integral mean of \( s(\phi_i) \), determining value \( \omega_1 \) is marked with \( c_\gamma \) in ISO standards and gear literature [1].

For resonance points location identification, symbol \( N = \omega_1/\omega_1 \) will be applied. As it is known from the theory of parametric vibrations [6], Eq. (3) gives resonance points not only at \( N = 1 \), but at points \( N = 1/k \) and \( N = 2/k \), \( k = 1, 2, 3, \ldots \), with decreasing on importance as \( k \) is increasing. In case of complex excitation, as in Eq. (2), besides the main excitation frequency
of $\omega$, other basic frequencies can be important, resulting in a complex dynamic response.

In case of load dependent mesh spring stiffness $s(\phi_1; \Delta \sigma)$, resonance locations being determined by that through $\omega$ will be load dependent as well. The $c_r$ in this case is load dependent as well; so for resonance locations symbol $\tilde{N}$ will be applied in cases, if single tooth pair deflection characteristics are of non-linear type.

5 GENERATION AND ANALYSIS OF INNER EXCITATION FUNCTION

The study of dynamic response of gear transmission is carried out by the application of the previously presented gear inner excitation model in case of a train with gears of number of teeth 53/65, and $m=12$. Two variants, one with normal tooth profiles, the other with long relief [7] were applied. For the probabilistic law of pitch error values, as for manufactured ones, Gaussian normal distribution was assumed. Random pitch error series was generated applying Monte-Carlo technique, under continuous control of the generated pitch error and accumulated pitch error values, corresponding to tolerance class DIN 7. In this way, one can avoid to obtain non-realistic gear models, and results. On Fig 5. and Fig. 6. pitch error values and accumulated pitch error functions are given, generated from Gaussian distribution function with zero expected value.

For avoiding inertia influence, rolling down simulations with $n_1=0$ can be carried out, with different $F_N$ values, by taking the corresponding outer torques on gears. In case of $F_N=0$, the contact function $\delta(\phi_1)$ is obtained. In this special case, transmission error function and kinematic excitation function are the same. On Fig. 7. $\delta(\phi_1)$ functions are represented for normal toothing and with tip relief, for two complete pinion rotation, realising the contact of $2.\phi_1=106$ different profile pair combinations. Fourier components $C_{\delta \phi}$ of transmission error curve of Fig. 7.a., i. e. for normal toothing, are presented on Fig. 8. One can see that besides the excitation
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Figure 8. Fourier components of the transmission error curve at $F_N=0$

Figure 9. Reduced stiffness curves at constant nominal load $F_N$, for gears with tip relief

Figure 10. Fourier components of the reduced stiffness function for gears with tip relief

As expected, important lower frequency components appear, due to the lower frequency pitch error and accumulated pitch error influence.

6 SIMULATED DYNAMIC TRANSMISSION ERROR CURVES

For representing the transmission error, the angular displacement error curves of the pinion (driver) were simulated, under steady state conditions, i.e. at rolling down with constant nominal speed and load. For the study of frequency response of the system, Fourier analyses of dynamic transmission error response curves were carried out. For single tooth force-deflection curve a non-linear one, with two variants of basic stiffness value [5,8], coded as WBHp for a bigger, and WBHKp for a smaller mean stiffness was applied.

Figure 11. and Fig 12. present mesh force dynamic factor curves, obtained by continuous rolling down simulations under
smooth acceleration, with increasing $n_1$ input speed and constant nominal tooth mesh forces respectively. These curves (with different scales in horizontal direction at different mesh forces) give a general orientation on highly different system behaviour under different nominal mesh forces on one side, and in cases of different toothing types, i.e. normal toothing Fig. 11. and that of with tip relief, Fig 12. on the other side. Differences between the two variants, the load influence and the load dependent, not monotonic response for case with tip relief are clearly to identify. Detailed analysis of dynamic factors is given in [10].

Figure 11. $K_D$ mesh force dynamic factors for gears with normal profiles

System behaviour with ideal toothing is not treated here, see e.g. in [9].

For frequency response analysis, continuous rolling down simulations were carried out at constant nominal mesh forces, and constant input speeds. These latter were selected in the resonance regions $\tilde{N} = 1/2$ and $\tilde{N} = 1$, for two complete rotation of the driver, involving the sequential contact of 106 tooth profile pair combinations, corresponding to the contact function on Fig. 7. Due to the fact that the gears are of random pitch errors, the simulated curves present irregular shape and can be considered as quasi-random realisation functions of a random process.

Figure 12. $K_D$ mesh force dynamic factors for gears with tip relief

Figure 13. Transmission error curves at $\tilde{N} = 1$, for gears with normal profiles
For the general overview of transmission error evolution, selected curves are represented, generated at speeds in regions of resonance points at $\tilde{N}=1$, Fig. 13. and 14. On horizontal axis nominal angular position (time) of the pinion (driver), and real angular position difference $\Delta\phi_1$, related to the nominal one, on the vertical axis are given. The two first harmonic components of transmission error curves were removed for the sake of the better identification of the differences.

As comparison basis, empirical density functions, obtained with function values at equidistant rotation angle, in step function form are given as well, representing well the scatter bands. The curve shapes and density functions reflect clearly the differences at different nominal mesh forces, and for the two types of profiles, with considerably bigger scatters at low load levels, for the profiles with tip relief.

7 FREQUENCY ANALYSIS OF TRANSMISSION ERROR CURVES

For analysing the frequency response of the system, Fourier analysis of the angular displacement error curves was applied. On Fig. 15. and 16, Fourier analysis results are presented in form of line spectra. The values correspond to different nominal loads at load dependent resonance points at $N=1/2$, at input speeds, given on the Figures. For allowing the treatment in a common basis, Fourier components $C_{\Delta\phi_1,k}$ are related to the basic component $C_{\Delta\phi_1}(F_N)$ at ordinal number corresponding to $f_z$, at nominal mesh force of 50N/mm, gear with tip relief. Components of the frequencies of $f_z$ and $2f_z$ are separately represented for the resonance points at $\tilde{N}=1/2$ on Fig. 17, and at $\tilde{N}=1$ (main resonance point) on Fig. 18. For sake of comparison, results obtained with errorless toothing are given as well.

In resonance points at $\tilde{N}=1/2$, expressed minimum region can be found, for the toothing with tip relief. Minimum values for teeth with pitch error are bigger as for errorless toothing, however the general curve behaviour is not changed. Optimum value is located at the tip relief design value. Starting from the optimum region in both directions, $f_z$ Fourier components increase, especially at lower nominal loads. Fourier components for normal toothing show a monotonic character, in wide region above the values of gears with tip relief.
Values for errorless tothing and with pitch error do not show remarkable differences.

The $2f_z$ values have an expressed minimum as well, at greater nominal loads as that of $f_z$. It is to remark, that the local maximum for errorless tothing (the origin of which can be explained with excitation intensity characteristics for errorless excitation function, not detailed here), disappear for gears with pitch error. At lower loads $2f_z$ components fall down.

In resonance points at $N=1$, more concentrated optimum regions were found, with more pronounced minimum values. The optimum point for teeth with pitch error is at a slightly lower nominal load, as in the errorless case. At low nominal loads the $f_z$ components are decreasing, because of the nonregular system response, see Fig. 11. and 12.

Comparing the Fourier spectra on Fig. 15. and 16, the importance of components on interval $1/2f_z...f_z$ at lower loads for teeth with tip relief is remarkable, tending to decrease with increasing load. However, at higher loads, components in frequency interval $f_z...2f_z$ are increasing. In case of normal tothing, components under frequency $f_z$ show only a slight variation. However, the components on greater frequencies are important on the whole load range, with relatively big values at $3f_z$. The region of the bigger frequencies in case of teeth with relief is characterised in general by small component values, predicting more convenient noise behaviour.

These results are in agreement with results of acoustic measurements in [11], showing similar tendencies for noise intensity in case of gears with tip relief.

8 CONCLUSIONS

For more realistic frequency response predictions in case of gear transmissions, taking into account randomly distributed pitch error series, methods are proposed for system modelling allowing the introduction and modelling the effects of manufacturing errors and tooth mesh specialities of any kind. For system response analysis, simulation results for driver movement was analysed, via Fourier analysis. Results have shown, that difference in system behaviour for gears with normal tothing and with tip relief can be detected and load dependence of system response can be followed. The Fourier spectra show, that one can predict, even in case of gears with manufacturing error, a more convenient system response in certain loads in case of gears with tip relief, i.e. better noise characteristics can be expected.
Figure 18. $f_z$ and $2f_z$, Fourier components vs. nominal load for normal toothing and with tip relief, in case of errorless teeth and with pitch error at $N=1$.

9 REFERENCES